

Unit - 3

Test of Hypothesis for Large Samples

- 1) Test of Hypothesis
- 2) Type I and Type II errors
- 3) Hypothesis testing concerning one mean and two means
- 4) Testing of Hypothesis for one proportion and difference of proportions.

Test of Hypothesis (T.O.H)

T.O.H is a statement about a parameters of population is tested for its validity or truthfulness.

Statistical Decisions (S.D)

S.D or conclusions about the population parameters on the basis of a Random sample from this population.

Statistical Hypothesis (S.H)

S.H is a assumption or guess about the parameter of population distribution.

- S.H consists of relationship between parameters of the population.

Null Hypothesis (N.H)

Is a S.H which is to be actually tested for acceptance or rejection.

- S.N.H is denoted by $N.H.H_0$:

Alternate Hypothesis (A.H)

Other than Null Hypothesis is called A.H and It is denoted by $A.H. H_1$

Test of Hypothesis: (Test of Significance)

It is a procedure to decide whether to accept or reject the Null Hypothesis ($N.H. H_0$)

Note:

Test of hypothesis decides whether a statement concerning a parameter is true or False instead of estimating the value of the parameter.

Errors in Sampling:

The main objective in sampling theory is to draw valid inference about the population parameters on the basis of sample results, as such we have 2 types of errors.

	H_0 is accepted	H_0 is rejected
H_0 is true	Correct Decision	Type - I error
H_0 is false	Type II error	Correct Decision

Type - I Error:

Reject $N.H$ when it is true

Probability of Type-I Error = α

Type - II Error:

Accept $N.H$ when it is False

Probability of Type-II Error = β

★ Level of Significance (α)

The probability of committing type-I error is called α .

- α is denoted by α

Note:

- 1) α measures the amount of risk associated in taking decisions.
- 2) If $\alpha = 5\%$, means, there are 5 chances in 100 that is N.H. H_0 is rejected (95% H_0 is accepted)
- 3) α is known as Size of the Test

Eq: N.H. $H_0 : \mu = \mu_0$

i.e., Population mean μ is equals to a specified constant μ_0

A.H. $H_1 : \mu > \mu_0$ (or) $\mu < \mu_0$ (or) $\mu \neq \mu_0$

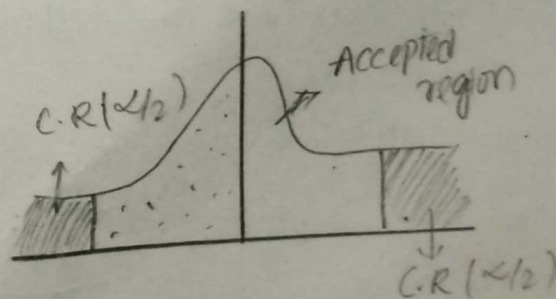
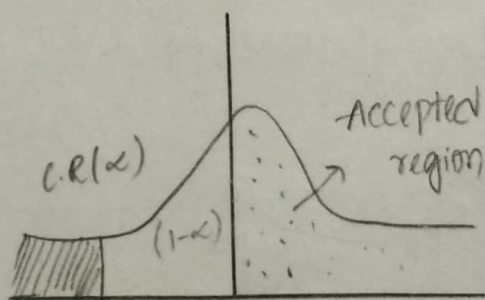
★ Critical Region (or) Rejected Region.

The Region of rejection of N.H

$\Rightarrow$ The area of Critical region

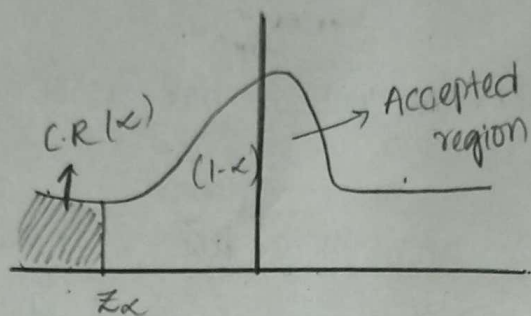
Note:

- 1) Critical region always lies on the tails of the distribution
- 2) Depending on the nature of A.H, C.R may lie on one side (or) both the sides of tails.



* Critical value (or) Significant Value.

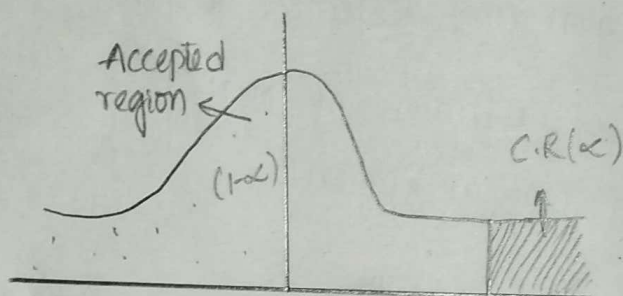
The value of test statistics which separates the critical region and accepted region is called Critical value (or) Significant value (z_α)



* Right One Tailed Test (R.O.T.T):

When the A.H.H₁ is of greater than type i.e., $A.H.H_1: \mu > \mu_0$ then the entire Critical region of area α lies on Right side tail of the probability density curve as shown in figure.

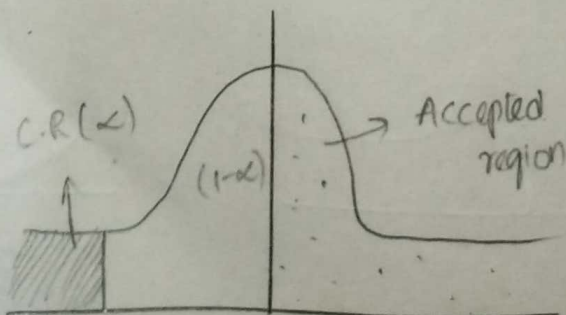
→ In such case the test of hypothesis is known as R.O.T.T



* Left One Tailed Test (L.O.T.T):

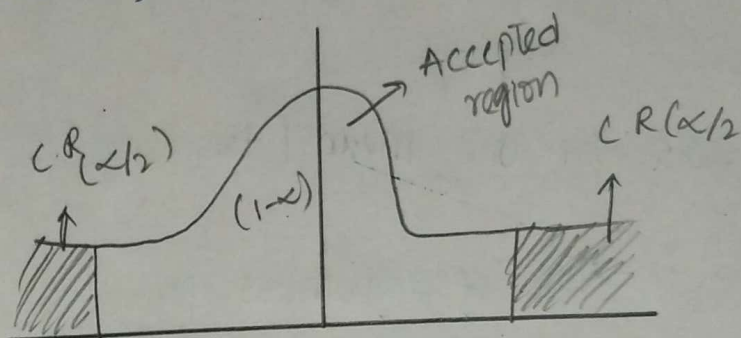
When A.H.H₁ is less than type i.e., $A.H.H_1: \mu < \mu_0$ then the entire critical region of area α , lies on left side tail of the probability density curve as show in figure.

→ In such case the test of hypothesis is known as L.O.T.T



* Two Tailed Test (T. T. T):

When A.H is of $\neq$ type i.e., $A.H.H_1: \mu \neq \mu_0$ then the entire C.R of area α lies on two tails of the probability density curve as shown.



→ In such case, the test of hypothesis is known as T. T. T.

Working Rule for test of Hypothesis.

- 1) Formulate $N.H. H_0$
- 2) Formulate $A.H. H_1$
- 3) Choose level of Significance α (if not given in prblm) $\alpha = 5\%$
- 4) C.R (or) Critical value (or) Tabulated z
Determine the C.R (or) Tabulated z based on Test
P.S. R.O.T.T, L.O.T.T and T.T.T
- 5) Test Statistic (or) calculated z :
Using the formula we obtain calculated z
- 6) Decision:
Accept / Reject the $N.H. H_0$ depending on Relation
btw Calculated z and Tabulated z

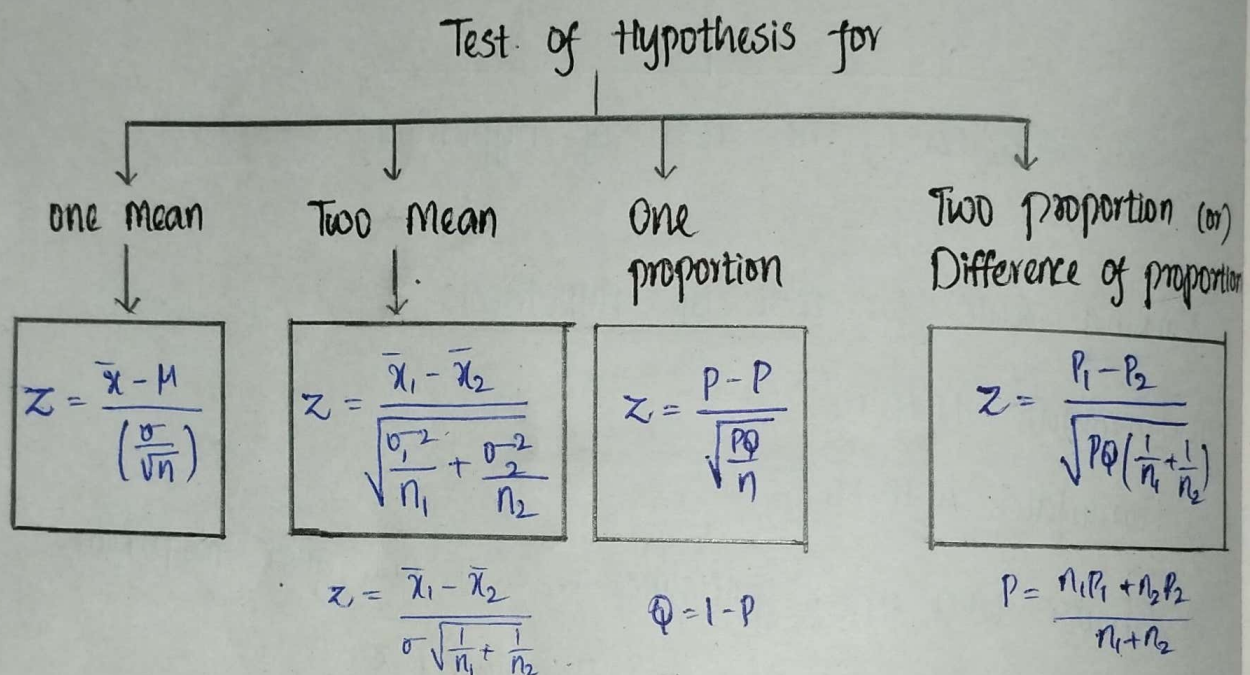
Note:

- 1) If $|\text{calculated } z| < \text{tab}^n z$, Accepted the $N.H. H_0$ is accepted
- 2) If $|\text{calculated } z| \geq \text{tab}^n z$, $N.H. H_0$ is rejected.

Note 1) For Large Samples ($n \geq 30$). If 'S' is given and σ is not given then S is replaced by σ

Note 2) If L.O.S (α) is not given, Assume it for Large Samples $\alpha = 5\%$. & for small samples ($\alpha = 1\%$) in general.

Test of Hypothesis for one mean [for Large Samples]



→ Let $\bar{x}$ be the Mean of a Random sample of size n drawn from a population

Then the test statistics for single population mean (μ) is given by

$$Z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

Using the above formula, we test the population mean (μ) is equal to specified Constant (μ_0) or not

i.e., N.H. $H_0 : \mu = \mu_0$

i) The length of life of Certain computers is approximately normally distributed with mean 800 hrs and S.D = 40 hrs. If the Random sample of 30 computers has an average life of 788 hrs, Test N.H that $\mu = 800$ hrs against the alternative that $\mu \neq 800$ hrs at

- i) 0.5% iv) 5% } accepted
 ii) 1% v) 10% }
 iii) 4% accepted vi) 15% L.O.S. rejected

Sol. Given, N.H. $H_0 : \mu = 800$

A.H. $H_0 : \mu \neq 800$

$$n = 30$$

$$\sigma = 40$$

$$\bar{x} = 788$$

i) N.H. $H_0 : \mu = 800$

2) A.H. $H_0 : \mu \neq 800$ ($\neq$ means T.T.T)

$$3) \text{ L.O.S } \alpha = 0.5\% \Rightarrow \frac{0.5}{100} \Rightarrow 0.005$$

4) C.R / C.V / tabulated z :

As A.H is of $\neq$ type, so use T.T.T)

$$\text{Since } \alpha = 0.005$$

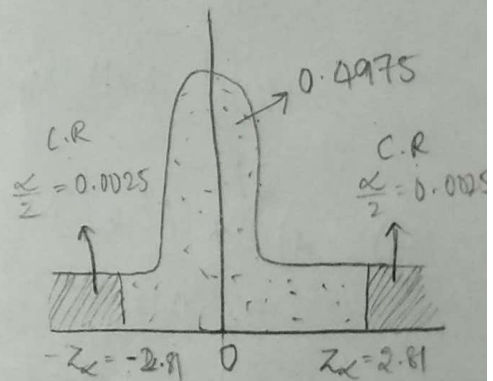
$$\frac{\alpha}{2} = \frac{0.005}{2} = 0.0025$$

From fig,

$$\begin{aligned} \text{Area from 0 to } z_{\alpha} &= 0.5 - 0.0025 \\ &= 0.4975 \end{aligned}$$

For the area 0.4975

$$\boxed{z_{\alpha} = 2.81}$$



5) Test Statistics / Calculated z :

Test Statistic for 1 mean is given by $z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$

$$= \frac{788 - 800}{\left(\frac{40}{\sqrt{30}}\right)} \Rightarrow \boxed{z = -1.64}$$

6) Decision:

Since $|\text{calculated } z| = |-1.64| = 1.64 < \text{Tabulated } z$

$$1.64 < 2.81$$

Hence N.H.H₀ is Accepted.

ii) 4) C.R / C.V / tabulated z :

As A.H is of $\neq$ type, use T.T.T

$$\text{Since } \alpha = 1\% = \frac{1}{100} = 0.01$$

$$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$$

From fig,

$$\begin{aligned} \text{Area from 0 to } z_{\alpha} &= 0.5 - 0.005 \\ &= 0.495 \end{aligned}$$

For the Area 0.495

$$\boxed{z_{\alpha} = 2.58}$$

6) Decision:

Since $|\text{cal. } z| = |-1.64| = 1.64 < \text{Tab. } z = 2.58$

Hence N.H.H₀ is Accepted.

iii) 4) C.R / C.V / tabulated z :

As A.H is of $\neq$ type, use T.T.T

$$\text{Since } \alpha = 4\% = \frac{4}{100} = 0.04$$

$$\frac{\alpha}{2} = \frac{0.04}{2} \Rightarrow 0.02$$

From fig

$$\begin{aligned} \text{Area from 0 to } z_{\alpha} &= 0.5 - 0.02 \\ &= 0.48 \end{aligned}$$

For the Area 0.48

$$Z_{\alpha} = 2.06$$

6) Decision:

$$\text{Since } |cal z| = |-1.64| = 1.64 < tab z = 2.06$$

$$1.64 < 2.06$$

Hence N.H.Ho is Accepted.

iv) 4) C.R / C.V / tabulated z :

As A.H is of $\neq$ type use T.T.T

$$\text{Since } \alpha = 5\% = \frac{5}{100} = 0.05$$

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

From fig,

$$\begin{aligned} \text{Area from 0 to } Z_{\alpha} &= 0.5 - 0.025 \\ &= 0.475 \end{aligned}$$

For the Area 0.475

$$Z_{\alpha} = 1.96$$

6) Decision:

$$\text{Since } |cal z| = 1.64 < tab z = 1.96$$

$$1.64 < 1.96$$

Hence N.H.Ho is accepted.

v) 4) C.R/C.V / tabulated z

As A.H is of $\neq$ type, use T.T.T

$$\text{Since } \alpha = 10\% = \frac{10}{100} = 0.1$$

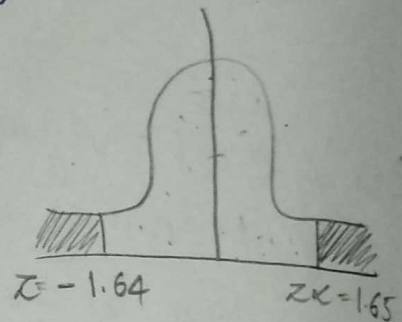
$$\frac{\alpha}{2} = \frac{0.1}{2} = 0.05$$

From fig,

$$\begin{aligned} \text{Area from } 0 \text{ to } z_{\alpha} &= 0.5 - 0.05 \\ &= 0.45 \end{aligned}$$

For the Area 0.45

$$\boxed{z_{\alpha} = 1.65}$$



6) Decision

$$\text{Since } |cal z| = 1.64 < tab z = 1.65$$

Hence N.H.H₀ is accepted.

vi) 4) C.R/C.V / tabulated z :

As A.H is of $\neq$ type, use T.T.T

$$\text{Since } \alpha = 15\% = \frac{15}{100} = 0.15$$

$$\frac{\alpha}{2} = \frac{0.15}{2} = 0.075$$

From fig,

$$\begin{aligned} \text{Area from } 0 \text{ to } z_{\alpha} &= 0.5 - 0.075 \\ &= 0.425 \end{aligned}$$

For Area 0.425

$$\boxed{z_{\alpha} = 1.44}$$

6) Decision

$$\text{Since } |cal z| = 1.64 < tab z = 1.44$$

$$1.64 < 1.44 \quad \times$$

Hence N.H.H₀ is rejected.

2) mice with an average life span of 32 months will live upto 40 months when fed by a certain nutritious food. If 64 mice fed on this diet have an average life span of 38 months and S.D of 5.8 months, is there any reason to believe that average life span is less than 40 months.

Sol.

Given, $M = 40$

$n = 64$

$\bar{x} = 38$

$s = 5.8$

$M = 40$
 $\bar{x} = 38$
 $s = 5.8$

i) N.H.H₀ $M = 40$

ii) A.H.H₀ $\mu < 40$ (L.O.T.T)

iii) Choose L.O.S $\alpha = 5\% \Rightarrow \frac{5}{100} = 0.05$

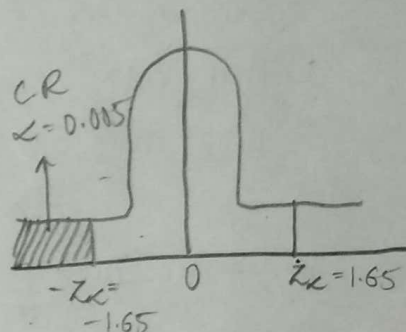
iv) C.R / C.V / tabulated z :-

As A.H is of $<$ type, use L.O.T.T

Since $\alpha = 0.05$

From fig

Area from 0 to $z_{\alpha} = 0.5 - 0.05$
 $= 0.005$



For Area 0.45

$z_{\alpha} = 1.65$

v) Test statistics / calculated z :

Test statistic for μ mean is given by $z = \frac{\bar{x} - M}{\left(\frac{\sigma}{\sqrt{n}}\right)}$

$\Rightarrow \frac{38 - 40}{\left(\frac{5.8}{\sqrt{64}}\right)} \Rightarrow \boxed{z = -2.75}$

vi) Decision:

Since $|cal z| = |-2.75| < tab z = 1.65 \Rightarrow 2.75 > 1.65$

Hence it is Rejected.

3) A machine runs on an avg of 125 hrs / year. A random sample of 49 machines has an annual avg use of 126.9 hrs with s.d 8.4 hrs. Does this suggest to believe that machines are used on the avg more than 125 hrs annually at 0.05 level of significance?

Sol. Given, $\bar{x} = 126.9$, $\sigma = 8.4$, L.O.S $\alpha = 0.05$
 $n = 49$, $M = 125$

i) N.H.H₀ ~~125~~ $M = 125$

ii) A.H.H₁ $M > 125$

iii) Choose L.O.S $\alpha = 0.05$

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

iv) C.R / C.V / tab z

As A.H is of $>$ type, use R.O.T.T

Since $\alpha = 0.05$

From fig,

$$\begin{aligned} \text{Area from 0 to } z_{\alpha} &= 0.5 - 0.05 \\ &= \cancel{0.05} 0.45 \end{aligned}$$

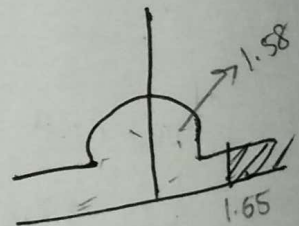
For Area 0.45

$$\boxed{z_{\alpha} = 1.65}$$

v) T.S / cal z

$$z = \frac{\bar{x} - M}{\left(\frac{\sigma}{\sqrt{n}}\right)} \Rightarrow \frac{126.9 - 125}{\left(\frac{8.4}{\sqrt{49}}\right)} \Rightarrow 1.58$$

$$\boxed{z = 1.58}$$



vi) Decision:

$$\text{Since } |cal z| = |1.58| < \text{tab } z = 1.65$$

Hence it is accepted.

4) A company claims that the mean thermal efficiency of diesel engines produced by them is 32.3%. To test this claim, a random sample of 40 engines were examined which showed, the mean thermal efficiency of 31.4% and S.D of 1.6%. Can the claim be accepted or not at 0.01 L.O.S?

Sol. Given, $\bar{x} = 31.4$, $\sigma = 1.6$, L.O.S = 0.01
 $n = 40$, $\mu = 32.3$

i) N.H.Ho $\mu = 32.3$

ii) A.H.Ho $\mu \neq 32.3$

iii) Choose L.O.S $\alpha = 0.01$

iv) C.R / C.V / tab z

As A.H is not given, use T.T.T

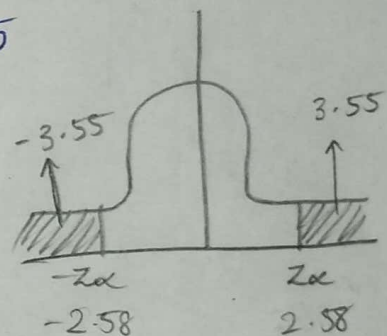
$$\text{Since } \alpha = 0.01 \quad \frac{\alpha}{2} = \frac{0.01}{2} = 0.005$$

From fig,

$$\text{Area from 0 to } z_{\alpha} = 0.5 - 0.005 = 0.495$$

For Area 0.495

$$\boxed{z_{\alpha} = 2.58}$$



v) T.S / cal z

$$z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{31.4 - 32.3}{\left(\frac{1.6}{\sqrt{40}}\right)}$$

$$\boxed{z = -3.55}$$

vi) Decision:

$$|cal z| < tab z$$

$$3.55 < 2.58 \Rightarrow 3.55 > 2.58$$

Hence N.H.Ho is Rejected.

5) To determine whether the mean breaking strength of synthetic fibre produced by a certain company is 8kg or not, a random sample of 50 fibres were tested yielding a mean breaking strength of 7.8 kg. If S.D is 0.5 kg test at 0.01 L.O.S.

Sol. Given, $\bar{x} = 7.8$, $\sigma = 0.5$, L.O.S = 0.01

$$n = 50 \quad M = 8$$

i) N.H.H₀ $M = 8$

ii) A.H.H₁ $M \neq 8$

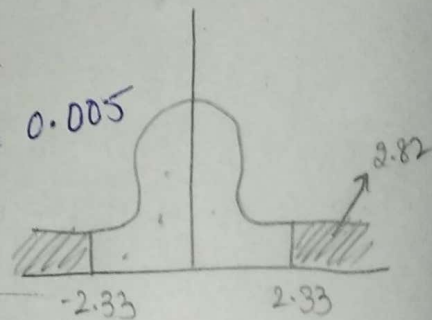
iii) choose L.O.S $\alpha = 0.01$

iv) C.R / C.V / tab z

$\neq$ ISO T.T.T type

Since $\alpha = 0.01$ $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

Area from 0 to $z_{\alpha} = 0.5 - 0.005$
 $= 0.495$



$$z_{\alpha} = 2.88 \quad z_{\alpha} = 2.58$$

v) cal z

$$z = \frac{\bar{x} - M}{\left(\frac{\sigma}{\sqrt{n}}\right)} \Rightarrow \frac{7.8 - 8}{\left(\frac{0.5}{\sqrt{50}}\right)} \Rightarrow$$

$$z = -2.82$$

vi) Decision

$$|cal z| < tab z$$

$$2.82 < 2.58 \Rightarrow 2.82 > 2.58$$

Hence it is rejected

6) Can it be concluded that the avg lifespan of Indian is more than 70 yrs if a random sample of 100 Indians has an avg lifespan of 71.8 yrs with S.D of 8.9 years

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Sol. Given, $n = 100$, $M = 70$
 $\bar{x} = 71.8$, $\sigma = 8.9$

i) N.H.H₀ : $\mu = 70$

ii) A.H.H₀ : $\mu > 70$

iii) $\alpha = 5\% = \frac{5}{100} = 0.05$

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

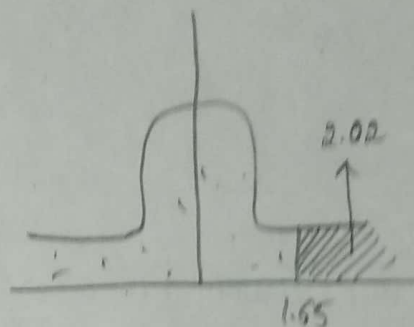
iv) C.R / C.V / tab z

> type, use R.D.T.T

$$\alpha = 0.05$$

$$\text{Area from 0 to } z_{\alpha} = 0.5 - 0.05 \\ = 0.45$$

$$z_{\alpha} = 1.65$$



v) cal z

$$z = \frac{\bar{x} - M}{\left(\frac{\sigma}{\sqrt{n}}\right)} \rightarrow \frac{71.8 - 70}{\left(\frac{8.9}{\sqrt{100}}\right)}$$

$$z = 2.02$$

vi) Decision - $|cal z| < tab \Rightarrow 2.02 < 1.65 \Rightarrow 2.02 > 1.65$

Hence N.H.H₀ is Rejected.

7) It has previously been recovered that the avg depth of ocean at a particular region is 67.4 fathoms. Is there reason to believe this at 0.01 $\alpha = 0.5$ if the readings at 40 random locations in that particular region showed a mean of 69.3 with S.D of 5.4 fathoms.

Sol. Given, $\alpha = 0.01$, $n = 40$

$$\bar{x} = 69.3, \sigma = 5.4, \mu = 67.4$$

i) N.H.H₀ : $\mu = 67.4$

ii) A.H.H₀ : $\mu \neq 67.4$

is

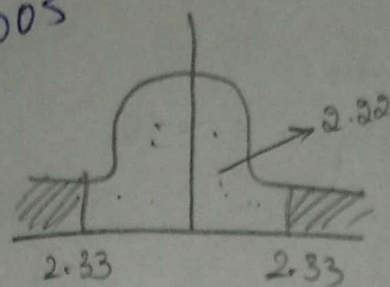
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iii) $\alpha = 0.01$

iv) tab z

$\neq$ T.T.T type
 $\alpha = 0.01 \quad \frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

Area from 0 to $z_{\alpha} = 0.5 - 0.005$
 $= 0.495$



$z_{\alpha} = 2.58$

v) cal z

$z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} \Rightarrow \frac{69.3 - 67.4}{\left(\frac{5.4}{\sqrt{40}}\right)} \quad z = 2.22$

vi) Decision $|cal z| < tab z \Rightarrow 2.22 < 2.58$

Hence N.H.H₀ is accepted

8) A company producing computers states that the mean lifetime of its computers is 1600 hrs. Test this claim at 0.01 & 0.5 against the A.H. $\mu < 1600$ hrs if 100 computers produced by this company has mean lifetime of 1570 hrs with S.D of 120 hrs.

Sol. Given, $\mu = 1600$, $\alpha = 0.01$

$\bar{x} = 1570$, $\sigma = 120$, $n = 100$

i) N.H.H₀ $\mu = 1600$

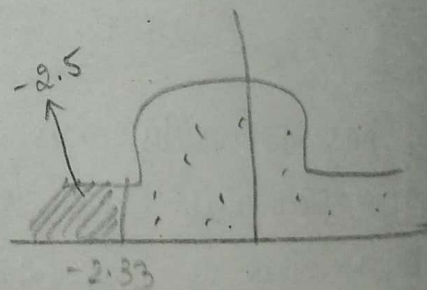
ii) A.H.H₁ $\mu < 1600$

iii) $\alpha = 0.01$

iv) tab z , use L.O.T.T

Area from 0 to $z_{\alpha} = 0.5 - 0.01$
 $= 0.49$

$z_{\alpha} = -2.33$



v) cal $z \quad z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} \Rightarrow \frac{1570 - 1600}{\left(\frac{120}{\sqrt{100}}\right)} \quad z = -2.5$

vi) Decision $|cal z| < tab z \quad 2.5 < 2.33 \Rightarrow 2.5 > 2.33$

Hence N.H.H₀ is Rejected

Test of hypothesis for two means.

When $\bar{x}_1$ be the mean of Random sample of size n_1 , drawn from a population A with mean μ_1 and variance σ_1^2 similarly;

let $\bar{x}_2$ be the mean of Random sample of size n_2 drawn from a population B with mean μ_2 and variance σ_2^2

The Test statistics for two mean is given by

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Here we want to test whether the 2 population means μ_1 and μ_2 are equal or not.

Here N.H. $H_0 : \mu_1 = \mu_2$

Note 1) For large samples, If s_1 and s_2 are given then they can be replaced by σ_1 and σ_2 .

2) If $\sigma_1 = \sigma_2 = \sigma$ then the test statistics formula is

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Problems

- i) In a Random sample of 100 tubelights produced by company A. The mean life time of tube light is 1190 hrs with S.D of 90 hrs. Also in a random sample of 75 tubelights from company B. The mean life time is 1230 hrs with S.D of 120 hrs. Is there a difference b/w a mean life times of the two brands of tubelight a significance level of

a) 0.05

b) 0.01.

50) Given $n_1 = 100$ $\bar{x}_1 = 1190$ $S_1 = 90 = \sigma_1$
 $n_2 = 75$ $\bar{x}_2 = 1230$ $S_2 = 120 = \sigma_2$

a) 1) N.H. $H_0 : \mu_1 = \mu_2$

2) A.H. $H_1 : \mu_1 \neq \mu_2$ (T.T.T)

3) L.O.S $\alpha = 0.05$

4) C.R / C.V / tab z

As A.H is $\neq$ Type so use T.T.T

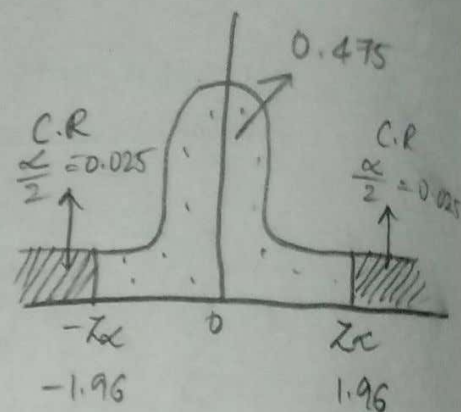
Since $\alpha = 0.05$

$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

From fig,

Area from 0 to $z_{\alpha} = 0.5 - 0.025$
 $= 0.475$

For Area 0.475 $\boxed{z_{\alpha} = 1.96}$



5) T.S / cal z

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \Rightarrow \frac{1190 - 1230}{\sqrt{\frac{90^2}{100} + \frac{120^2}{75}}} \Rightarrow -2.42$$

$\boxed{z = -2.42}$

6) Decision:

$|tab\ z| < |cal\ z| < tab\ z$

$1.96 < |-2.42| < 1.96 \Rightarrow 2.42 < 1.96 \Rightarrow 2.42 > 1.96$

Hence, it is Rejected.

i) b) $\alpha = 0.01$

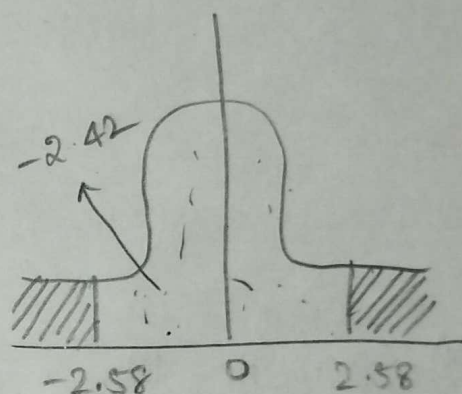
4) Since $\alpha = 0.01$

$$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$$

From fig

Area from 0 to $z_{\alpha} = 0.5 - 0.005$
 $= 0.495$

For Area 0.495 $\boxed{z_{\alpha} = 2.58}$



5) cal z

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \Rightarrow \frac{1190 - 1230}{\sqrt{\frac{90^2}{100} + \frac{120^2}{75}}} \quad \boxed{Z = -2.42}$$

6) Decision: $|cal z| < tab z \Rightarrow 2.42 < 2.58$

Hence N.H.H₀ is accepted.

Assignment

- 4) The means of two large samples of size 1000 and 2000 members are ~~67.5~~ 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from same population of standard deviation 2.5 inches use $\alpha = 0.05$.

Sol.

Given.

$$n_1 = 1000$$

$$\bar{x}_1 = 67.5$$

$$\sigma_1 = \sigma_2 = \sigma = 2.5$$

$$n_2 = 2000$$

$$\bar{x}_2 = 68.0$$

$$\alpha = 0.05$$

i) N.H.Ho $\mu = \mu_2$

ii) A.H.Ho $\mu \neq \mu_2$

iii) $\alpha = 0.05$

iv) tab z

$$z_{\alpha} = 1.96$$

v) cal z

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \Rightarrow \frac{\bar{x}_1 - \bar{x}_2}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \Rightarrow \frac{675 - 68}{2.5 \sqrt{\frac{1}{1000} + \frac{1}{2000}}}$$

$$z = -5.16$$

vi) Decision:

$$|cal z| < tab z$$

$$5.16 < 1.96 \Rightarrow 5.16 > 1.96$$

Hence N.H.Ho is Rejected.

2) Given, $\bar{x}_1 = 496.31$, $\sigma_1^2 = 17.18$, $n_1 = 60$
 $\bar{x}_2 = 485.41$, $\sigma_2^2 = 14.73$, $n_2 = 30$

a) $\alpha = 0.05$

b) $\alpha = 0.01$

i) N.H.Ho $M_1 = M_2$

ii) A.H.Ho $M_1 \neq M_2$

iii) L.O.S $\alpha = 0.05$

iv) tab z

$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

Area from 0 to $z_\alpha = 0.5 - 0.025$
 $= 0.475$

$z_\alpha = 1.965$

v) cal z

$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \Rightarrow \frac{496.31 - 485.41}{\sqrt{\frac{(17.18)^2}{60} + \frac{(14.73)^2}{30}}}$

$z = 3.12$

vi) Decision: |cal z| < tab z

$3.12 < 1.96 \Rightarrow 3.12 > 1.96$

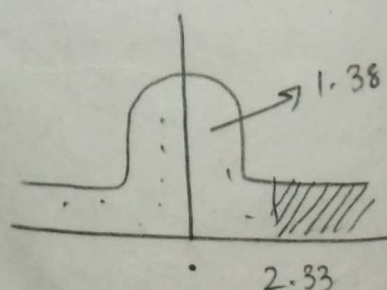
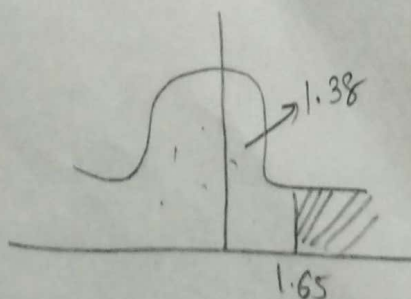
Hence N.H.Ho is rejected.

3) Given, $n_1 = 40$, $\bar{x}_1 = 647$, $\sigma_1 = 27$
 $n_2 = 40$, $\bar{x}_2 = 638$, $\sigma_2 = 31$

a) 0.05 } L.O.S

b) 0.01 }

Superior >



b) i) N.H.Ho $M_1 = M_2$

ii) A.H.Ho $M_1 \neq M_2$

iii) L.O.S $\alpha = 0.01$

iv) tab z

$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

Area from 0 to $z_\alpha = 0.5 - 0.005$
 $= 0.495$

$z_\alpha = 2.33$

v) cal z

$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

$= \frac{496.31 - 485.41}{\sqrt{\frac{(17.18)^2}{60} + \frac{(14.73)^2}{30}}}$

$z = 3.72$

vi) Decision: |cal z| < tab z

$3.12 < 2.33$

Hence N.H.Ho rejected

- a) i) N.H.Ho $\mu_1 = \mu_2$
 ii) A.H.H_a $\mu_1 > \mu_2$
 iii) $\alpha = 0.05$
 iv) tab z

Area from 0 to z_α
 $0.5 - 0.05$
 $= 0.45$

$z_\alpha = 1.65$

v) cal z

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\Rightarrow \frac{647 - 638}{\sqrt{\frac{(27)^2 + (31)^2}{40}}}$$

$z = 1.38$

vi) Decision $|cal z| < tab z$

$$1.38 < 1.65$$

Accepted
 Hence it is ~~Rejected~~

b) i) N.H.Ho $\mu_1 = \mu_2$

ii) A.H.H_a $\mu_1 > \mu_2$

iii) $\alpha = 0.01$

iv) tab z

Area from 0 to z_α
 $0.5 - 0.01$
 $= 0.49$

$z_\alpha = 2.33$

v) cal z

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{647 - 638}{\sqrt{\frac{(27)^2 + (31)^2}{40}}}$$

$z = 1.38$

vi) Decision $|cal z| < tab z$

$$1.38 < 2.33$$

Hence it is ~~rejected~~
 Accepted

4) Given, $n_1 = 1600$ $\bar{x}_1 = 172$

$\sigma_1 = 6.3 \rightarrow$ men

$n_2 = 6400$ $\bar{x}_2 = 170$

$\sigma_2 = 6.4 \rightarrow$ women

$$\alpha = 0.01$$

men taller than women
 $\mu_1 > \mu_2$

i) N.H.Ho $\mu_1 = \mu_2$

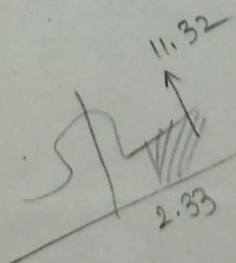
ii) A.H.H_a $\mu_1 > \mu_2$

iii) $\alpha = 0.01$

iv) tab z

Area from 0 to $z_\alpha = 0.5 - 0.01$
 $= 0.49$

$z_\alpha = 2.33$



v) cal z $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

$$= \frac{172 - 170}{\sqrt{\frac{(6.3)^2}{1600} + \frac{(6.4)^2}{6400}}}$$

$z = 11.32$

vi) $|cal z| < tab z$

Hence rejected

5) Given, $n_1 = 50$ $\bar{x}_1 = 68.2$ $\sigma_1 = 2.5$ — Boys
 $n_2 = 50$ $\bar{x}_2 = 67.5$ $\sigma_2 = 2.8$ — Girls

Age of boys heavier than girls
 $\text{boys} > \text{girls}$

L.O.S a) 0.05 b) 0.01

- i) N.H.Ho $\mu_1 = \mu_2$
 ii) A.H.Ho $\mu_1 > \mu_2$
 iii) L.O.S $\alpha = 0.05$
 iv) Area from 0 to Z_α

$0.5 - 0.05$
 $= 0.45$

$Z_\alpha = 1.65$

v) cal z

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}}$$

$$Z = \frac{68.2 - 67.5}{\sqrt{\frac{(2.5)^2 + (2.8)^2}{50}}}$$

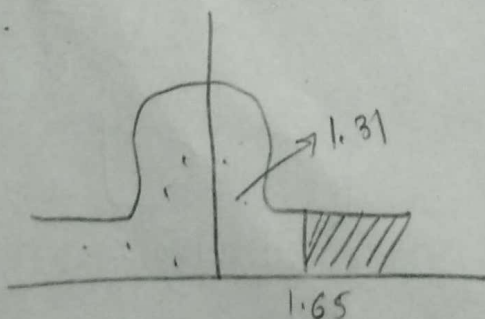
$Z = 1.31$

vi) Decision

$|cal z| < tab z$

$1.31 < 1.65$

Hence N.H.Ho is
 Accepted



i) N.H.Ho $\mu_1 = \mu_2$

ii) A.H.Ho $\mu_2 > \mu_1$

iii) L.O.S $\alpha = 0.01$

iv) tab z

Area from 0 to Z_α

$0.5 - 0.01$
 $= 0.49$

$Z_\alpha = 2.33$

v) cal z

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}}$$

$$= \frac{68.2 - 67.5}{\sqrt{\frac{(2.5)^2 + (2.8)^2}{50}}}$$

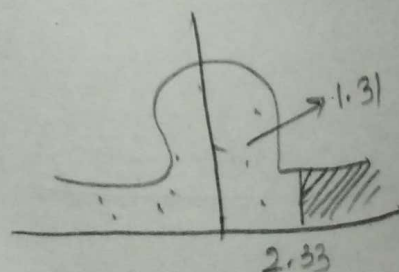
$Z = 1.31$

vi) Decision

$|cal z| < tab z$

$1.31 < 2.33$

Hence N.H.Ho is
 Accepted



Test of Hypothesis for One proportion

Suppose, a large sample of size 'n' is drawn from a Normal population. To test the significant difference b/w sample proportion (p) and population proportion (P), We use the Test statistics

$$Z = \frac{P - P_0}{\sqrt{\frac{PQ}{n}}}$$

where $Q = 1 - P$

Here, N.H. $H_0: P = P_0$ (A specified constant P_0)

- i) Experience has shown that 20% of a manufacture product is of the top quality. In one day's production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 level.

Sol. Given, $P_0 = 20\%$, $p = 50/400$
 $= \frac{20}{100} = 0.2$

$$Q = 1 - P \Rightarrow 1 - 0.2 \Rightarrow 0.8$$

$$n = 400$$

$$\text{Let } P = \frac{50}{400} = \frac{1}{8} = 0.125$$

i) N.H. $H_0: P = 20\% = 0.2$

ii) A.H. $H_1: P \neq 0.2$ (T.T.T)

iii) $\alpha = 0.05$

iv) C.R / C.V / tab Z

A.H. H_1 is $\neq$ type, use T.T.T

Since $\alpha = 0.05$

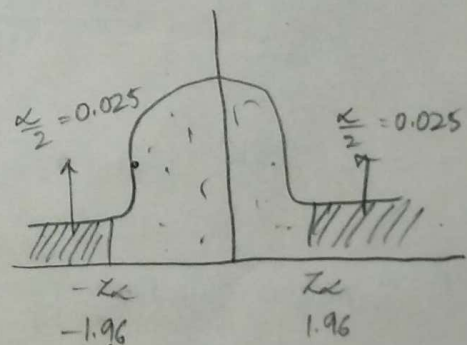
$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

From fig,

$$\begin{aligned} \text{Area from 0 to } Z_{\alpha} &= 0.5 - 0.025 \\ &= 0.475 \end{aligned}$$

For the Area 0.475

$$Z_{\alpha} = 1.96$$



v) T.S / cal z:

Test Statistics for one proportion is given by,

$$Z = \frac{P - P}{\sqrt{\frac{PQ}{n}}} \Rightarrow \frac{0.125 - 0.2}{\sqrt{\frac{0.2 \times 0.8}{400}}} \Rightarrow -3.75$$

$$Z = -3.75$$

vi) Decision:

$$|cal Z| < tab Z$$

$$3.75 < 1.96$$

Hence N.H.H₀ is Rejected.

Test of Hypothesis for Difference of proportion [2 proportions]

Suppose, two large samples of sizes n_1, n_2 are taken respectively from two different populations. To Test the significant difference b/w sample proportions (p_1 and p_2) The Test Statistics is given by

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\text{where } P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} \text{ and } Q = 1 - P$$

Here N.H.H₀ $P_1 = P_2$

- i) Random Samples of 400 men & 600 women were asked whether they would like to have a flyover near their residence. 200 men & 325 women were in favour of the proposal are same at 5% level

Sol. Given $n_1 = 400$
 $n_2 = 600$

$$\text{let } P_1 = \frac{200}{400} \Rightarrow 0.5, \quad P_2 = \frac{325}{600} = 0.54$$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} \Rightarrow \frac{400 \times 0.5 + 600 \times 0.54}{400 + 600}$$

$$P = 0.524$$

$$Q = 1 - P$$

$$= 1 - 0.524$$

$$Q = 0.476$$

i) N.H.H₀ $P_1 = P_2$

ii) A.H.H₁ $P_1 \neq P_2$

iii) $\alpha = 0.05$ $\alpha = 5\% = \frac{5}{100} = 0.05$

iv) C.R / C.V / tab Z

A.H.H₁ is $\neq$ type, use T.T.T

Since $\alpha = 0.05$

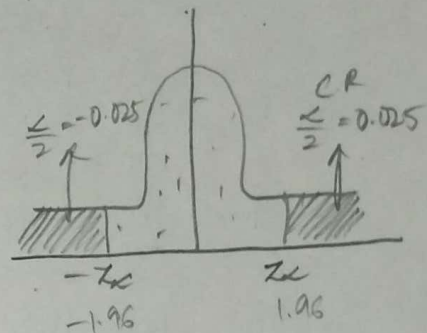
$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

Area from 0 to $Z_{\alpha} = 0.5 - 0.025$

$$= 0.475$$

From table Area of 0.475

$$Z_{\alpha} = 1.96$$



v) T.S / cal Z

Test statistics for two proportions is given by

$$Z = \frac{P_1 - P_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \Rightarrow \frac{0.5 - 0.54}{\sqrt{(0.524 \times 0.476 \left[\frac{1}{400} + \frac{1}{600} \right])}}$$

$$Z = -1.24$$

vi) Decision:

$$|cal Z| = |-1.24| = 1.24 < tab Z = 1.96$$

Hence N.H.H₀ is Accepted.

4) On the basis of their total scores, 200 candidates of a civil service examination are divided into two groups, the upper 35% and the remaining 65%. Consider the first question of the examination. Among the first group, 40 had the correct answer. Whereas among the second group, 80 had the correct answer. On the basis of these results, can one conclude that the first question is not good at discriminating ability of the type being examined here?

Sol. Let $n_1 = 35\%$ of 200 = $\frac{35}{100} \times 200 = 70$ } 200

$n_2 = 65\%$ of 200 = $\frac{65}{100} \times 200 = 130$

Let $p_1 = \frac{40}{70} = 0.57$

$p_2 = \frac{80}{130} = 0.61$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} \Rightarrow \frac{70 \times 0.57 + 130 \times 0.61}{70 + 130}$$

$P = 0.59$

$Q = 1 - P \Rightarrow 1 - 0.59$

$Q = 0.41$

i) N.H. $H_0: P_1 = P_2$

ii) A.H. $H_1: P_1 \neq P_2$

iii) Choose $\alpha = 0.05$ $\alpha = 5\% = 0.05$

iv) C.R / C.V / tab Z

A.H is $\neq$ type, use T-T-T

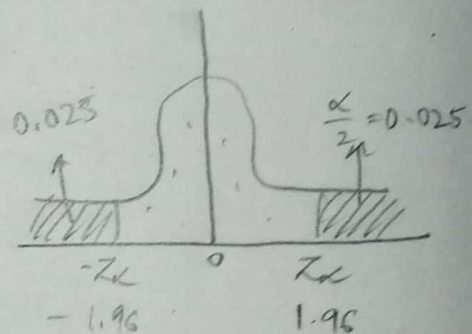
Since $\alpha = 0.05$

$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

From fig,

Area from 0 to $Z_{\alpha} = 0.5 - 0.025$
 $= 0.475$

$Z_{\alpha} = 1.96$



v) cal z

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \Rightarrow \frac{0.67 - 0.57}{\sqrt{(0.6 \times 0.4)\left(\frac{1}{60} + \frac{1}{140}\right)}}$$

$$\boxed{Z = 1.32}$$

vi) Decision.

$$|cal z| < tab z$$

$$1.32 < 1.96$$

Hence N.H.Ho is Accepted.